2012 Summer School on Geometry and Data

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Washington State University
and the
Center for Geometric Analysis and Data

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1 Introduction

This summer school, which will run from July 9 through July 27, in the Pullman-Moscow area on the border of Idaho and Washington, will bring students to the leading edge of research at the intersection of geometric measure theory and geometric analysis with data analysis and inference from noisy data. Our overall goal is to attract students to this area of research by involving them in both theoretical and computational aspects.

There are four subthemes that we will explore, along with example questions motivating our studies:

1. **Geometric Inference under Uncertainty** How can we infer tangent planes from noisy samples of a manifold? What is the dimension of those tangent planes? What is the curvature of a set that we know only from noisy samples? How can we stabilize boundary computations to the effects of uncertainty?

2. **Graphs and Analysis in Metric Spaces** How much of the analysis we know for $\mathbb{R}^n$ can we carry over to graphs (and more generally, to metric spaces)? What methods are there for reducing dimension of data sets which live on graphs? What are graph wavelets and how can we use them? What is “diffusion geometry”? 
3. **Approximation and Denoising** Given a noisy image or shape, how much of this noise can we remove? Is there a way to exploit geometric insights to improve approximation methods? Can we exploit the graph structure inherent in many data sets to remove noise?

4. **Extraction of Information from Data** What tools from geometric analysis can we exploit to see more deeply into data sets? What do measure theoretic tools like densities tell us about sets in $\mathbb{R}^2$ and $\mathbb{R}^3$?

These will be explored in the lectures and in the computational labs. The lectures will be aimed at advanced undergraduates and beginning graduate students and will be designed to inspire and illuminate. The 3-4 daily lectures will be divided between supporting material and expositions of a list of 19 papers that are listed below. These ideas will be illustrated and explored computationally in the afternoon sessions.

Evenings and weekends will be filled with time to interact socially and chances to explore the beautiful natural setting of the northwest, with one extended four day excursion to Lake Coeur d’Alene. Cafe excursions, picnics, hikes, and more will mix naturally with extended discussions of the problems and ideas inspired by the talks and computational sessions.

## 2 Scientific Program Details

The summer school will revolve around the exposition and exploration of ideas contained in a set of papers listed in the next subsection. Some of these papers are long and complicated, yet each of them can be explained intuitively and understood through a careful, guided exploration. And that is what we will do. Also, there are a great number of ideas that can be experimented with computationally, which is what we will be doing in the afternoons, under the guidance of Asaki, Sottile, Krishnamoorthy and Scofield.

### 2.1 Courses

**Papers** Expositions based on Papers that are listed in the next subsection. All the important pieces will be explained so that (1) you understand the statement of the results, (2) what insights go into proving things and (3) what is useful for other work and applications to data problems. **Lecturers: Everybody.**

**Data Lab** Computational Data Labs These will occupy the afternoons and will explore computationally, the content of the lectures that occupy the mornings. **Lecturers: Asaki, Sottile, Krishnamoorthy and Scofield.**

**GMT** Geometric Measure Theory: Introduction and Customized Synopsis The exposition will use intuitive explanations and lots of pictures – because we can and because this is the most efficient way to communicate the subject. **Lecturers: Taylor, Snipes, Camfield, Vixie, Meadows.**

**AHA** Harmonic Analysis: Wavelets and all that Various pieces of applied harmonic analysis are used in some of the papers we will be exploring this summer. A short synopsis of wavelets will be given. **Lecturers: Hammond, Le, Maggioni, Szlam.**

**Compute** Computational Data Analysis These are companion lectures to the Data Labs above, and will be used to rapidly bring everybody up to speed on a minimalist set of ideas used in those labs. **Lecturers: Asaki, Sottile, Krishnamoorthy and Scofield, Allard.**
Graphs  What you need to know about Graphs and Analysis on Graphs  Just as it sounds, here we will distill out the parts of the analysis of graphs, and on graphs, that we will need.

Lecturers: Sottile, Hammond, Asaki, Scofield, Krishnamoorthy, Szlam, Maggioni, Allard

2.2 Papers

**Allard**  Allard et al. *Multiscale Geometric Methods for Data Sets II: Geometric wavelets* [1]

**Asaki**  Asaki et al. *Image Denoising by Regularization on Characteristic Graphs* [2]

**Camfield**  Camfield and Snipes, *Notes on Analysis in Metric Spaces* [3]

**Chazal**  Chazal et al., *Stability of Boundary Measures* [4]

**Coifman**  Coifman and Lafon, *Diffusion Maps* [5]

**Federer**  Federer, *Curvature Measures* [6]

**Hammond**  Hammond et al. *Wavelets on Graphs via Spectral Graph Theory* [7]

**Ibrahim1**  Ibrahim et al. *On the uniqueness of non-asymptotic densities for simple closed curves* [8]

**Ibrahim2**  Ibrahim et al. *Simplicial Flat Norm with Scale* [9]

**Kaslovsky**  Kaslovsky and Meyer, *Optimal Tangent Plane Recovery From Noisy Manifold Samples* [10]

**Larson**  Larson and Vixie, *Notes on Doubling measures* [11]

**Le**  Le and Memoli, *Local scales on curves and surfaces* [12]

**Lerman**  Lerman et al., *Robust computation of linear models, or How to find a needle in a haystack* [13]

**Little**  Little et al. *Multiscale geometric methods for data sets I: Estimation of Intrinsic Dimension* [14]

**Meadows**  Meadows and Vixie, *Notes on Reach and Curvature Measures* [15]

**Morgan**  Morgan and Vixie, *L1TV computes the Flat Norm* [16]

**Taylor**  Taylor and Meyer, *A random walk on image patches* [17]

**Vixie**  Vixie et al., *Multiscale flat norm signatures for shapes and images* [18]

**Zhang**  Zhang et al., *Hybrid Linear Modeling via Local Best-fit Flats* [19]

Here is a listing of which papers go with each of the themes. (Most of these papers fit in more than one section, even though we have classified only a few below as multitheme papers.)

**Inference**  Geometric Inference under Uncertainty

*  Kaslovsky, Little, Chazal
*  Federer, Lerman, Zhang
*  Allard, Meadows
2.3 Lecturers

There will be 17 faculty who lecture and assist students in understanding and computing and exploring.

**Kevin R. Vixie** Before moving to WSU in 2008, Kevin started a team (guerrilla warfare style) at Los Alamos National Laboratory that focused on the exploitation of mathematical insights for the analysis of tough data challenges. Before that, he was a graduate student with Andrew M. Fraser and worked on dynamical systems and various signal analysis problems. His area of expertise is geometric measure theory and its application to data problems. Here is a link to his website.

**Mauro Maggioni** Mauro is on the faculty at Duke University where he works on a variety of problems from pure and applied harmonic analysis, analysis of large data sets, and a rich variety of applications. Before joining the faculty at Duke, he spent time at Yale and Washington University (where he did his graduate work with Guido Weiss). Mauro, who can justify vacations to Italy in many different ways, can be researched a bit more by going here.

**Francois Meyer** Francois is an Electrical Engineer, though really an applied mathematician in disguise (as all good engineers are), and is a professor at the University of Colorado. He works on a variety of subjects, all of them interacting with data and most of them having to do with some aspect of applied harmonic analysis. Here is a link to his website.

**Thomas J. Asaki** Tom is a physicist turned mathematician and algorithmic wizard with a particular interest in optimization and data analysis. Tom helped start the Los Alamos team that vixie established, after deciding to join the team Vixie was in at Los Alamos. In addition to his video game fame, Tom is known for his “Cities Puzzle” and his exquisite lecturing style. You can find out more about this Montana’n in exile by visiting here.

**William K. Allard** Bill Allard was a student of Fleming’s and also worked with Federer – for example, reading and editing (in detail) Federer’s *Geometric Measure Theory* in his spare time, (yes, as a graduate student). After time at Princeton, where he wrote his famous paper on varifolds, he settled at Duke university where he has been ever since. As long as you don’t ask him what he thinks about Matlab, he will be happy to tell you about his forays into the intricacies of scientific programing and the use of scripting languages (like Matlab) to
solve data analysis challenges. He continues to work in data analysis and geometric measure theory, which you can figure out from visiting here.

Jean Taylor  Jean is an expert in geometric measure theory and its intersection with materials science. She is pretty good at understanding the bubbles that others blow, and can even show you a beautiful Scientific American article that she wrote with her PhD advisor, Fred Almgren. In addition to her deep work in geometric measure theory, she is known for her travels and her interest in hiking and walking (she walks around Manhattan Island every now and then). She is now Emeritus from Rutgers University and spends time at the Courant Institute, not far from where she lives, when not traveling. You can find out a bit more here.

Marie Snipes  Marie worked with Juha Heinonen and Mario Bonk at the University of Michigan, extending geometric measure theory ideas to Banach Spaces. Before that, she was a student at Harvey Mudd in that large city called Southern California, just after which, she was an analyst in the Air Force, where she developed mathematical models and won an award for having the most organized desk. (Seriously – it was so extreme her desk made the commanding general’s meeting agenda one week.) Naturally enough this (and other talents) obtained for her a professorship at Kenyon College where she does research and teaches bright undergraduates to choose the right path in life, which includes cutting bagels properly, playing with soap films, and learning to appreciate (tolerate) geeky math jokes. For more information, you can go here.

Chris Camfield  Chris is currently a professor at Hendrix College, where he carries the standard for geometric measure theory and especially its generalization to metric spaces. Chris is a superb lecturer and an enthusiastic promoter of undergraduate research. And he is a former running star – lucky for those of us who might want to run with him from time to time, he is not as fast as he used to be. Here is his webpage.

Triet Le  When he is not rock climbing, Triet is thinking about mathematics or taking pictures he then analyzes, sometimes with help of a beer or two. He liked Southern California so much that he got both his BS and PhD at UCLA, but then decided that a bit of climate change was in style, so he moved to Yale (where else?) to work with that mathematical troubadour, Peter Jones. The work in Local Scales that he did with Peter is the basis of some of what he will talk about here, but the second installment of that work – his work with F. Memoli, listed above – is what will be looked at most closely. For more on him and his work and interests you can go here.

Daniel Kaslovsky  Daniel is just graduating from University of Colorado and will be moving to Yale for a postdoc at the end of the summer. Before that he got a degree in computer science and then took the next natural step and worked in sports management, but tiring of the money (and everything that goes with that), he decided that mathematics was a good compromise between computer science and big sports. Having lived in NM, he also knows what the question Red or Green? means – do you? To know more, you will have to come and listen to him explain his compelling dissertation work or nose around his website here.

Arthur Szlam  Arthur, who worked with Raphy Coifman at Yale (our lecturer sample sets the probability of working with, or in the vicinity of, Coifman at about 30% – but then you know what they say about statistics), works in applied and computational harmonic analysis. He is currently in New York City – well actually, he is currently in residence at the IMA in
Minnesota for the *Mathematics of Information* year. His work can be explored a bit more by going here and nosing around.

**David Hammond**  David Hammond began life thinking he might be a chemist, but eventually realized that that wasn’t what one did if you wanted to live a long time. So he did the logical thing and decided on a career in mathematics. After a stimulating 4 year stint at Caltech, he decided he needed to see the world a bit and joined the Peace Corps, after which he worked with Simoncelli at NYU. In addition to inventing graph wavelets and exploiting mathematical insights to understand data, he plays classical guitar to mushrooms. To verify that and other tidbits, you can visit his website here.

**Bala Krishnamoorthy**  Bala, known for his expertise in optimization (and rock climbing), also works in applied algebraic topology. Applying his mathematical insights to a wide variety of problems from other areas has gained him a diverse repertoire and an aptitude for finding something productive to do in many different problem contexts. As a result of coauthoring the paper with Ibrahim and Vixie on the simplicial flat norm, he is currently looking at computational aspects of geometric measure theory conjectures involving flat norms. For more you can go here.

**Matthew Sottile**  Matt, famous for his patience with institutional bureaucracy, has successfully left institutions like Los Alamos and University of Oregon to become a senior scientist at Galois in Portland, Oregon, where he uses much of his background in computer science, mathematics, statistics, large scale computing and data analysis to solve a large variety of problems. Or at least he uses a subset to solve a subset. Currently, he is exploring ways in which that company can move in the direction of Big Data, in line with his history as a leading member on the team that Vixie started at Los Alamos. He will be attending for multiple reasons, one of which is to spot possible hot shots to recruit for his company’s ventures into big data. You can find out more (and less) by visiting his website here.

**Thomas Scofield**  Thomas, who is currently at Calvin College in Michigan, is interested in evangelizing gifted undergraduates with the news that research in pure and computational analysis is fun, and is where lots of the action is. When he is not tap dancing, he is teaching or thinking about the intersection of analysis and data. He is also known for innovative solutions to optical problems (enough said about that). For more information, see his website here.

**Alex Meadows**  Alex, who can stand close to Rick Schoen without melting, was a student of Leon Simon’s (who was also Rick’s advisor). Alex can be forgiven his obsession with geometric measure theory and related subjects since he was subjected to the early influences of Frank Morgan, who was a professor of his at Williams College, where Alex was an undergraduate. But, in his world where donkeys and hyperdo’s and tornados mix, one can never be bored, which can be proven by going here. (Five extra points to anyone who finds the earliest known connection between KRV and AM.)

**Linh Nguyen**  After getting his Bachelor’s degree in Vietnam, Linh moved to Texas where he worked on PDEs and inverse problems, eventually getting his PhD with Kutchment at Texas A&M. After visiting Berkeley, he joined the faculty at University of Idaho. There he continues his work on inverse problems and has added a new thread in image analysis with supporting threads of some aspects of geometric measure theory. When he is not zipping back and forth between WSU and UI, you might find him visiting friends in Texas or collaborators in Seattle.
or California. But if you too are curious about Linh’s connection to Arnaldo Pomodoro, you will have to start solving this mystery by visiting Linh’s website here.

2.4 Computational Labs

The labs will be focused on exploring as much of the lecture material as possible through Matlab experiments that the students will carry out on their own laptops.

More detail will be added as the time for the summer school gets closer.

3 Environment and Activities

The summer school will be held in Moscow, Idaho a few miles from Pullman, Washington. University of Idaho is located in Moscow, while Washington State University is located in Pullman. There is a walking/running/biking trail that connects the two towns. Moscow Mountain, five miles from Moscow has numerous hiking and biking trails.

The weekend of July 14-15, there will be hikes and picnics as well as impromptu games and get together’s at local coffee shops and restaurants.

We are planning a four day excursion, July 19-22, to Lake Couer d’Alene Idaho. The lake is a particularly beautiful and quiet lake, which is quite deep (over 1000’ deep) and large (over 140 miles of coastline. Camping, boating, hiking, swimming, and backpacking are among the activities that participants can choose from while there.

More details will be available as we get closer to the summer school session.

4 Logistics

The summer school will run from Monday, July 9, through Friday, July 27. Financial support for 20 students is available, covering travel (up to 500$), lodging and a modest per deim of 20$ per day. In addition to the breakfast provided by the hotel, a catered lunch will be provided to all the summer school participants.

4.1 Applying for Admission and for Financial Support

Applications for admission should be sent by email to vixie@speakeasy.net with the subject line “CGAD 2012 Summer School Application:<your name>”. We are expecting to financially support 20 graduate (or advanced undergraduate) students. Students who can support themselves will also be admitted, but must apply for admission.

The Application should contain, or have attached:

1. A current CV.

2. Names of 2 references who you have asked to send letters for you. (Ask them to email the references to vixie@speakeasy.net with the subject line “Recommendation for:<your name>”.)

3. A statement of why you want to attend, what background you have that equips you to take advantage of the material to be discussed in this summer school.
4. copy of your transcripts (unofficial copy is OK).

5. **Very important**: indicate whether or not you are applying for financial support. We have money to cover 20 students.

**Deadline:** we will fill the spots as we find excellent candidates, so send your applications in as soon as you can. We expect to have all the financial support given away by the last week of March, but if you find this after that date, you can always apply to come at your own expense.

### 4.2 More Details

More details will be provided in the near future. Keep checking:

http://geometricanalysis.org/Workshops/2012SummerSchool

for updates to this and other documents.

**References**


